LEI DE GAUSS E FLUXO ELÉCTRICO

GAUSS'S LAW AND ELECTRIC FLUX

1) $\vec{E} = \dfrac{Q}{4\pi\varepsilon R^2}\vec{a}_R$ $\varepsilon = \varepsilon_0$ $\oint \vec{D}\cdot d\vec{S} = Q_{int}$ $\vec{D} = \varepsilon\vec{E}$

A) Recorrendo da relação entre densidade de fluxo eléctrico e campo eléctrico, determine a correspondente expressão da densidade de fluxo eléctrico. Using the relation between electric flux density and electric field, find the corresponding expression to determine the density of electric flux.

$\vec{D} = \varepsilon\vec{E}$ $\vec{D} = \dfrac{Q}{4\pi\varepsilon R^2}\vec{a}_R\,\varepsilon$ $\vec{D} = \dfrac{Q}{4\pi R^2}\vec{a}_R$

Resposta: Answer: $\vec{D} = \dfrac{Q}{4\pi R^2}\vec{a}_R$

B) Recorrendo da definição da lei de Gauss, mostre que a carga que provoca o campo é a própria carga Q. Using the definition of Gauss's law, show that the electrical charge causing the electrical field is the electrical charge Q itself.

$\oint \vec{D}\cdot d\vec{S} = Q_{int}$ $\oint \dfrac{Q}{4\pi R^2}\vec{a}_R \cdot d\vec{S} = Q_{int}$ $d\vec{S}_{\phi\theta} = R^2\sin(\theta)d\theta d\phi\,\vec{a}_R$

$$\int_0^{2\pi}\int_0^{\pi}\frac{Q}{4\pi R^2}\vec{a}_R\cdot R^2\sin(\theta)d\theta d\phi\,\vec{a}_R = Q_{int}$$

$$\int_0^{2\pi}\int_0^{\pi}\frac{Q}{4\pi}\sin(\theta)d\theta d\phi = Q_{int}$$

$$\frac{Q}{4\pi}\int_0^{2\pi}\int_0^{\pi}\sin(\theta)d\theta d\phi = Q_{int}$$

$$\frac{Q}{4\pi}\int_0^{2\pi}\left[-\cos(\theta)\right]_0^{\pi}d\phi = Q_{int}$$

$$-\frac{Q}{4\pi}\int_0^{2\pi}\left[\cos(\theta)\right]_0^{\pi}d\phi = Q_{int}$$

$$-\frac{Q}{4\pi}\int_0^{2\pi}(-1-1)d\phi = Q_{int}$$

$$2\frac{Q}{4\pi}\int_0^{2\pi}d\phi = Q_{int}$$

$$2\frac{Q}{4\pi}\left[\phi\right]_0^{2\pi} = Q_{int}$$

$$2\frac{Q}{4\pi}(2\pi-0) = Q_{int}$$

Resposta: Answer: $Q = Q_{int}$

2) $\varepsilon = \varepsilon_0$ $\qquad \vec{\mathrm{E}} = \dfrac{r^3 \rho_v}{3\varepsilon_0 (r+R)^2} \vec{a}_R; \quad R \geq 0$ $\qquad \vec{\mathrm{D}} = \varepsilon \vec{\mathrm{E}}$ $\qquad \oint \vec{\mathrm{D}} \cdot d\vec{\mathrm{S}} = Q_{\mathrm{int}}$

A) Recorrendo da relação entre densidade de fluxo eléctrico e campo eléctrico, determine a correspondente expressão da densidade de fluxo eléctrico. Using the relation between electric flux density and electric field, find the corresponding expression to determine the density of electric flux.

$$\vec{\mathrm{D}} = \varepsilon \vec{\mathrm{E}} \qquad \vec{\mathrm{D}} = \frac{r^3 \rho_v}{3\varepsilon_0 (r+R)^2} \vec{a}_R \, \varepsilon_0 \qquad \vec{\mathrm{D}} = \frac{r^3 \rho_v}{3(r+R)^2} \vec{a}_R$$

B) Recorrendo da definição da lei de Gauss, encontre a expressão da carga total armazenada na esfera. Using the definition of Gauss's law, find the expression of the total electrical charge stored in the sphere.

$$\vec{\mathrm{D}} = \frac{r^3 \rho_v}{3R_R^2} \vec{a}_R; \quad R_R \geq r \qquad \oint \vec{\mathrm{D}} \cdot d\vec{\mathrm{S}} = Q_{\mathrm{int}} \qquad \oint \frac{r^3 \rho_v}{3R_R^2} \vec{a}_R \cdot d\vec{\mathrm{S}} = Q_{\mathrm{int}}$$

$$d\vec{S}_{\phi\theta} = R_R^2 \sin(\theta) d\theta d\phi \, \vec{a}_R \qquad \int_0^{2\pi} \int_0^{\pi} \frac{r^3 \rho_v}{3R_R^2} \vec{a}_R \cdot R_R^2 \sin(\theta) d\theta d\phi \, \vec{a}_R = Q_{\mathrm{int}}$$

$$\frac{r^3 \rho_v}{3} \int_0^{2\pi} \int_0^{\pi} \sin(\theta) d\theta d\phi = Q_{\mathrm{int}} \qquad \frac{4\pi}{3} r^3 \rho_v = Q_{\mathrm{int}}$$

Resposta: Answer: $\dfrac{4\pi}{3} r^3 \rho_v = Q_{\mathrm{int}}$